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CASCADE TRANSFER OF ENERGY, VORTICITY, AND A PASSIVE IMPURITY IN HOMOGENEOUS ISOTROPIC TURBULENCE (TWO- AND THREE-DIMENSIONAL)

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UDC 532.517.4

Unidimensional turbulence is modeled experimentally in flows behind a grid. An extensive amount of empirical data has been accumulated on this subject, but several problems arise in connection with its analysis. Of primary interest is the reason that different exponents n in exponential laws describing the decay of fluctuation energy $\langle u^2\rangle \sim t^{-n}$ are obtained in different experiments (see [1], for example). It is believed that these differences are connected with the "initial" conditions (although the authors of [2] assert that the problem lies in the analysis itself). The more refined spectral characteristics of the velocity field (and the field regarding a passive impurity) also differ in different experiments [1, 3].

Recent investigations have also been concerned with quasi-two-dimensional turbulence realized (as hypothesized) in grid flows of a strongly conducting fluid in a strong transverse magnetic field [4-6]. The results of the experiments conducted here are also conflicting.

It is generally held that the first question that needs to be answered satisfactorily is the connection between the spectral characteristics (exponential asymptotes) and the exponent n in the exponential law of fluctuation energy decay $\langle u^2\rangle$. The point is that $\langle u^2\rangle$ is coarser than the spectrum, an experimental characteristic. Thus, its measurements are more reliable.

To establish such a connection, it is necessary to go outside the framework of the scale-invariant interval because, in the integral $\langle u^2\rangle = \frac{2}{3}\int_0^\infty E(k)\,dk$, the range of values of k for which E(k) is known should be broad enough to obtain a good approximation of the entire integral [3]. As a result, it is necessary to determine additional features of the process of vortex breakup (combination).

A vortex of a certain scale can be subdivided into two, three, or more smaller vortices. We will assume that for each fixed scale (wave number k) there is a certain probable multiplicity of subdivisions α_k. Meanwhile, the smaller vortices into which the initial vortex is subdivided are of approximately the same dimensions. Since the process of vortex breakup (combination) occurs as a result of inertial effects, we will assume that the inertial interaction of the vortices is realized mainly during the subdivision (combination). As a result of formalization of this physical hypothesis, we obtain an equation for the spectral function of the velocity field which accounts for the spectral hypotheses of Kolmogorov–Obukhov (for three-dimensional turbulence) and Kraichnan–Batchelor (for two-dimensional turbulence). The

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present investigation is devoted to study of the solutions of this equation and comparison of their properties with known grid experiments.

We will similarly use the Obukhov–Corrsin spectral hypothesis for the cascade transport of a passive impurity (in particular, temperature). It wil be shown that if the exponential spectral asymptotes of the velocity field and the impurity coincide, then the decay laws for $\langle u^2\rangle$ and $\langle\Theta^2\rangle$ also coincide. A correspondence with the experimental data is also established here.

1. Cascade Equation. The equation for the spectral fucntion of the velocity field F(k, t) can be formally written in the form [3]

$$\partial F(k,\ t)/\partial t = \Gamma(k,\ t) - 2\nu k^2 F(k,\ t). \tag{1.1}$$

It is customary to refer to Γ(k, t) as the rate of spectral energy redistribution; it is a function of k and t and a functional of F and is determined by inertial transport processes. If F is sufficiently small, then we expand Γ into a functional series in powers of F. Due to the nonlinearity of the inertial processes, this will be a series in fractional powers of F (an expansion in the neighborhood of a branch point):

$$\Gamma = \sum_{n=0}^{\infty} \int_0^{\infty} dk_1 \ldots dk_n F^{1/m}(k_1) \ldots F^{1/m}(k_n)\, G(k_1, \ldots, k_n; k, t)$$

(m is a fixed integer). Since the inertial effects in the Navier–Stokes model are quadratic with respect to velocity, we choose the leading degree in this expansion on the basis of correspondence with the Navier–Stokes model in the form

$$\Gamma = \int_0^{\infty} dk_1\, dk_2\, dk_3\, G(k_1, k_2, k_3; k, t)\, F^{1/2}(k_1)\, F^{1/2}(k_2)\, F^{1/2}(k_3). \tag{1.2}$$

The function G describes the inertial effect of vortices with the scales $k_1^{-1}, k_2^{-1}, k_3^{-1}$ on vortices with the scales k^{-1}.

Let us examine the case when this inertial effect is determined mainly by breakup (combination) of vortices. The breakup (combination) of eddies is characterized primarily by the multiplication factor, i.e., by the mean number of eddies formed after the breakup of one eddy (or after combination, conversely).

If the vortex subdivides into N vortices, then the multiplication factor of the subdivision α = 1/N (for the combination of N vortices into one vortex, α = N). We will assume that for a fixed value of k, there is a most probable value of α_k. If the inertial interaction of eddies is determined by their breakup (combination) over a sufficiently large interval of wave numbers, then the function $G(k_1, k_2, k_3; k, t)$ can be approximated by a singular generalized function with a carrier at the point $k_1 = k_2 = k_3 = \alpha k$. It follows from the well-known theorem on the structure of such functions [7] that $G(k_1, k_2, k_3; k, t)$ is uniquely represented in the form

$$G(k_1, k_2, k_3; k, t) = \sum_{|p| \leqslant N} g_p(k, t)\, D^p \delta(\alpha_k k - k_1)\, \delta(\alpha_k k - k_2)\, \delta(\alpha_k k - k_3),$$

where $g_p(k, t)$ is a function of k and t; N is the order of the function G; δ(x) is the Dirac delta function; $D^p f(\mathrm{x}) = \partial^{|p|} f(x)/\partial x_1^{p_1} \partial x_2^{p_2} \partial x_3^{p_3}$, $|p| = p_1 + p_2 + p_3$. The order of the function G is determined by the differential properties of the function F [7, p. 22]. Inserting this representation into (1.2), we obtain

$$\Gamma = \sum_{m=0}^{N} f_m(k, t)\, (-1)^m \alpha^{-m} \partial^m F^{3/2}(\alpha k, t)/\partial k^m \tag{1.3}$$

(f_m is a function of k and t). The order of function G can be determined by means of the Batchelor–Proudmen analysis from [8]. Without going into the details of this process, we note that N ≥ 2 for three-dimensional isotropic turbulence and N = 2 for the two-dimensional case.

As regards the coefficient functions $f_m(k, t)$, we will approximate $f_m(k, t)$ exponentially to ensure the existence of scale-invariant asymptotes for F(k, t): $f_m(k, t) = -c_m t^\gamma k^\delta$ (c_m, γ and δ are real parameters). The terms in the sum (1.3) with order m > 0 are important only for large-scale fluctuations. Thus, instead of (1.3), we write the approximation

$$\Gamma = -c_0 t^{\gamma} k^{\delta} F^{3/2}(\alpha k,\ t). \tag{1.4}$$

It is obvious that Eq. (1.4) is valid only on a certain interval of wave numbers, since it relates only to energy sinks.

Thus, we have

$$\partial F(k,\ t)/\partial t = -c_0 t^{\gamma} k^{\delta} F^{3/2}(\alpha k,\ t) - 2\nu k^2 F(k,\ t). \tag{1.5}$$

"Closing" hypothesis (1.4) is of fundamental importance for obtaining this equation, since it reflects the representation both of the cascade and of the physical process of eddy breakup.

2. Energy Cascade in the Three-Dimensional Case and Initial Conditions. What should we choose as an initial condition for Eq. (1.5)? The simplest would be the condition $F_0(k)$ = const, i.e., an equal distribution of the density of kinetic energy over the space of the wave vectors [3, p. 649]. Since the choice of initial condition will generally be determined by the specific physical situation, the adequacy of the initial condition chosen for the calculation should be checked by comparing the results of the solution with the corresponding experiment.

If we ignore the direct effect of viscosity on the interval of wave numbers being examined, then (1.5) is written as

$$\partial F(k,\ t)/\partial t = -c_0 t^{\gamma} k^{\delta} F^{3/2}(\alpha k,\ t). \tag{2.1}$$

On the interval of wave numbers on which α_k can be approximately assumed to be constant, we make the following substitution of variables:

$$\tau = c_0 t^{\gamma+1} k^{\delta}. \tag{2.2}$$

Then

$$\partial F(k,\ \tau)/\partial \tau = -F^{3/2}(\alpha k,\ \alpha^{\delta}\tau). \tag{2.3}$$

In the similarity case

$$dF(\tau)/d\tau = -F^{3/2}(\alpha^{\delta}\tau), \tag{2.4}$$

i.e., F(k, t) depends on k and t only in terms of the complex τ (2.2).

Let us find the dependence of $\langle u^2 \rangle$ on t. To do this, in the interval $\langle u^2\rangle \sim \int_0^\infty E(k)\,dk$, we replace the varible k by τ (2.2). Then

$$\langle u^2 \rangle \sim t^{-n},\ n = 3(1+\gamma)/\delta. \tag{2.5}$$

The constants γ and δ are determined from the Kolmogorov–Obukhov spectral hypothesis. The solution of Eq. (2.4) has the exponential asymptote ($\tau > 0$)

$$F(\tau) \sim \tau^{-2}, \tag{2.6}$$

from which the spectral energy density is

$$E(k,\ t) \sim t^{-2(1+\gamma)} k^{-2(\delta-1)}. \tag{2.7}$$

The Kolmogorov–Obukhov hypothesis yields the following on the inertial interval [3]:

$$E \sim \varepsilon^{2/3}(t)\, k^{-5/3}, \tag{2.8}$$

where

$$\varepsilon = d\,\langle u^2 \rangle/dt. \tag{2.9}$$

We use (2.5)-(2.9) to obtain

$$\delta = 11/6,\ \gamma = -4/15. \tag{2.10}$$

Inserting (2.10) into (2.5), we find

$$\langle u^2 \rangle \sim t^{-1.2}. \tag{2.11}$$

Thus, the Kolmogorov–Obukhov inertial hypothesis (2.8) corresponds to a decay of fluctuation energy of the form (2.11) if $F_0(k)$ = const is taken as the initial condition.

Uberoi was evidently the first to observe a relation of the form (2.11) [10] in an experiment behind a grid. Figure 1 shows values of the exponent n obtained in different grid

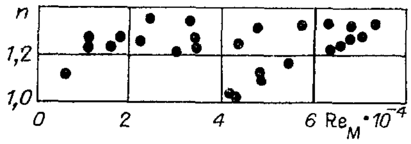

Fig. 1

experiments (the data was taken from [1]). It is evident that although most of the values for n differ little from 1.2, this value cannot be considered unique. Thus, it is currently not possible to empirically confirm the premise being discussed (Saffmen, having obtained a similar result by a different method, believes that the grid experiment data offers support for a connection between a "−5/3" law and n = 1.2).

3. Entropy Cascade in Two-Dimensional Turbulence. In contrast to three-dimensional turbulence, in the two-dimensional case the determining factor is not the energy cascade, but the entropy cascade (see [10, 11], for example). In this case, the Kraichnan–Batchelor hypothesis [10, 11] replaces the Kolmogorov–Obukhov inertial hypothesis:

$$E(k, t) \sim \varepsilon_\omega^{2/3} k^{-3}, \tag{3.1}$$

where $\varepsilon_\omega \sim d\Omega/dt$; $\Omega = \langle\omega^2\rangle/2$ is the mean square of the curl (entropy). If we now use the approach in Part 2, then we find for the initial condition $F_0(k)$ = const (equal distribution of energy in the wave-vector space) that exponential spectral asymptote (3.1) corresponds to the fluctuation energy decay law $\langle u^2\rangle \sim t^{-1}$, while for the initial condition $E_0(k)$ = const (equal distribution with respect to the scales) the asymptote corresponds to the law $\langle u^2\rangle \sim t^{-2/3}$.

Here, it turns out that

$$E(k, t) \sim t^{-2} k^{-3} \tag{3.2}$$

in either case.

It is assumed that at low values of the magnetic Reynolds number, quasi–two-dimensional turbulence is realized in flows of strongly conducting fluid in a strong transverse magnetic field [4-6, 10, 11]. Here, the role of the magnetic field is reduced to creating and suppressing two-dimensional turbulence in the plane normal to the induction vector. The field does not interact with the two-dimensional turbulence itself [11] which is very convenient for experimental study of such turbulence − since the magnetic field does not affect its properties in the given case. Figure 2 shows experimental data [4] obtained in a flow of mercury behind a grid with B = 0 and 0.68T (i.e., without a magnetic field and in a strong magnetic field). A pronounced exponential asymptote $E_1(k) \sim k^{-5/3}$ was observed in the experiment with B = 0, while an equally pronounced asymptote $E_1 \sim k^{-3}$ was seen in the experiment with B = 0.68T. Figure 3 also shows [4] the empirically established dependence of E_1 on t (compare with Eq. (3.2), t = x/U, where U is the mean velocity of the flow behind the grid and M is the mesh of the grid). It must be noted that, compared to Part 2, the agreement between theory and experiment here is considerably better (points 1 and 2 correspond to k_0 = 1240 and 2530 m^{-1}). Similar empirical results (n = 1, 2 and $E_1 \sim k^{-5/3}$ at B = 0, n = 2/3 and $E_1 \sim k^{-3}$ at B = 0.8T) were obtained in experiments of the same type (mercury behind a grid in a transverse magnetic field) in [5] (evidently the first to conduct such tests) and in [6]. It is not clear why the agreement between theory and experiment is very good in these studies and is not in most of the experiments in Part 2. It may be that the only difference between the experiments in Part 3 and Part 2 is the fluid (mercury). It must be emphasized that both initial conditions F_0 = const and E_0 = const are realized in the two-dimensional case (i.e., n = 1 and 2/3; see Fig. 2). This also confirms the experimental data reported in [12], where two values of n, depending on the initial conditions, were obtained in a grid flow of mercury in a transverse magnetic field: n = 1 and 2/3.

4. Cascade Transport of a Scalar Impurity. The breakup of eddies leads to the breakup of discontinuities of a scalar impurity (in particular, temperature [3, 13]). The authors of [13, 14] developed an approach to cascades in a scalar impurity field that is similar to the Kolmogorov–Obukhov approach to cascades in a velocity field. Extending this concept here to a scalar impurity, we obtain an equation analogous to (2.1) for the three-dimensional spectrum of the field of an impurity Θ:

$$\partial F_\Theta(k, t)/\partial t = -c_0 t^\nu k^\delta F^{1/2}(\alpha k, t) F_\Theta(\alpha k, t). \tag{4.1}$$

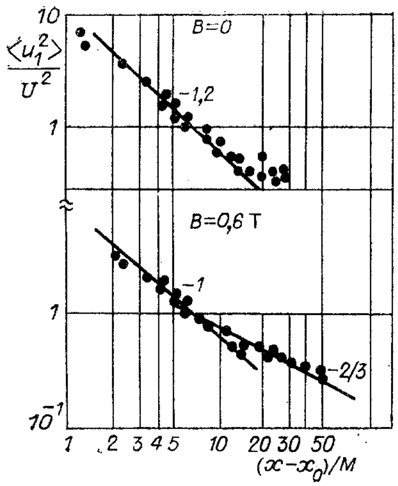

Fig. 2

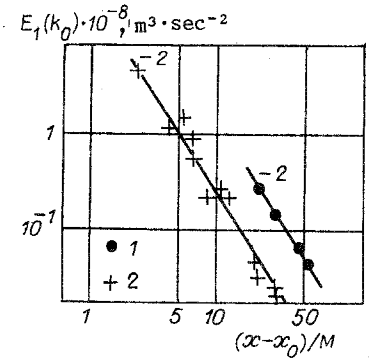

Fig. 3

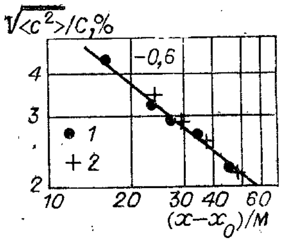

Fig. 4

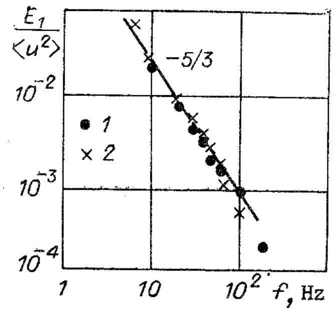

Fig. 5

If the parameters γ and δ in (4.1) are the same in (2.1), then we can use the Obukhov–Corrsin hypothesis for the inertial-convective interval [3] $E_\Theta \sim \bar{N}\varepsilon^{-1/3}k^{-5/3}$, as in Part 2, to obtain $\langle u^2\rangle \sim t^{-1.2}$ and $\langle\Theta^2\rangle \sim t^{-1.2}$ from (2.1) and (4.1) (with the initial conditions $F_0(k)$ = const and $F_\Theta(k, 0) = \text{const}$) .

Figure 4 (taken from [15]) shows experimental data on the evolution of fluctuations of a scalar impurity (salinity) behind a grid. Also shown are experimental results from [16] on the evolution of fluctuations of temperature (as the scalar impurity) behind a grid (point 1 corresponds to $\sqrt{\langle c^2\rangle}/C$, while point 2 corresponds to $\sqrt{\langle\Theta^2\rangle}/\Delta T_0$). The kinetic energy of the fluctuations decreases in accordance with the same law as the mean square of the impurity. Figure 5 (taken from [15]) shows experimental data on unidimensional longitudinal fluctuations of velocity and concentration spectra (point 1 corresponds to $G(k_1)/\langle c^2\rangle$, while 2 corresponds to $E_1(k_1)/\langle u^2\rangle$) .

It should be noted that the decay of fluctuations of a passive impurity behind grids is very sensitive to the "initial" conditions, which seriously complicates the problem of analyzing results obtained in different experiments (see [17, 18], for example).

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SOUND PROPAGATION IN POLYDISPERSED GAS SUSPENSIONS

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UDC 534.2:532.529

The majority of studies of acoustics of gas suspensions have investigated propagation of linear and slightly nonlinear waves in monodispersed mixtures [1-5]. The effect of polydispersion on propagation of linear monochromatic waves was first studied in [6]. However only the simple case of low mass content of the suspended phase was considered, in which case the contribution of particles of a given size to sound dispersion and dissipation is actually proportional to their mass fraction in the mixture. The present study will investigate unique features of sound wave propagation in polydispersed gas and vapor suspensions for arbitrary (not necessarily small) mass content of suspended particles or droplets for the first time. Some of the results were reported previously in [7].

1. General Considerations. Real gas suspensions of both natural and artificial origin are usually not monodispersed. They contain particles of quite differing sizes, which often differ greatly from each other. The dispersion composition of such mixtures can be characterized at each point in space by a particle distribution function over size $N(a, \mathbf{r}, t)$, as well as the minimum $a_{\min}(\mathbf{r}, t)$ and maximum $a_{\max}(\mathbf{r}, t)$ radii. We have

$$dn(a, \mathbf{r}, t) = N(a, \mathbf{r}, t)\,da, \quad n(\mathbf{r}, t) = \int_{a_{\min}}^{a_{\max}} N(a, \mathbf{r}, t)\,da.$$

Here a is the particle radius, $\mathbf{r}$ is the radius vector of the point, t is time, dn is the number of particles per unit volume having radii from a to a + da, n is the total number of particles of all sizes per unit volume of mixture at the space-time point $(\mathbf{r}, t)$.

We will consider the quite general case of a mixture with phase transitions at phase separation boundaries. In the process of motion of such a mixture, the particle (droplet) dis-

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